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WADC TECHNICAL REPORT 54-250
PART 8
ASTIA DOCUMENT No. AD 97265

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# DYNAMIC SYSTEM STUDIES: RECORDERS

*C. DONALD LaBUDDE*

*UNIVERSITY OF CHICAGO*

(187)

*SEPTEMBER 1956*

WRIGHT AIR DEVELOPMENT CENTER

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# DYNAMIC SYSTEM STUDIES: RECORDERS


*C. DONALD LaBUDDE*

*UNIVERSITY OF CHICAGO*


*SEPTEMBER 1956*

AERONAUTICAL RESEARCH LABORATORY
CONTRACT AF 33(038)-15068, SUPPLEMENTS 2 AND 11
PROJECT 7060

WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO


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## FOREWORD

The Advisory Board on Simulation has concluded a three-year research program in air weapon system dynamics sponsored by Wright Air Development Center, with P. W. Nosker/WCRR as project engineer. This volume is one of the following 16 comprising the final report, WADC TR 54-250, entitled Dynamic System Studies:

| Part No. | Subtitle | Editing Agency |
|---|---|---|
| 1 | Conclusions and Recommendations | University of Chicago |
| 2 | The Design of a Facility | " " " |
| 3 | The Mission of a Facility (Confidential) | " " " |
| 4 | Technical Staff Requirements | " " " |
| 5 | Analog Computation | Naval Ordnance Lab. |
| 6 | Operation & Maintenance Procedures for Analog Computers | University of Chicago |
| 7 | Digital Computers | " " " |
| 8 | Recorders | " " " |
| 9 | Flight Tables (Confidential) | " " " |
| 10 | Performance Requirements for Flight Tables | Mass. Inst. of Tech. |
| 11 | Load Simulators (Confidential) | Cook Research Lab. |
| 12 | Guidance Simulation **(Secret)** | Naval Ordnance Lab. |
| 13 | Error Studies | University of Chicago |
| 14 | Error Analysis for Differential Analyzers (written by F. J. Murray, Columbia U., and K. S Miller, N.Y.U.) | " " " |
| 15 | Air Vehicle Characteristics (Secret) | " " " |
| 16 | Aerodynamic Studies (written by M. Z. Krzywoblocki, U. of Ill.) | " " " |

The history of the project and a complete bibliography may be found in Part 1. All reports may be obtained through the project engineer.

This report represents the culmination of the assignment to determine the proper mission, equipmentation, operating procedures, and personnel for an engineering facility in the field of air weapon systems dynamics. The subdivisions of the report correspond to these four basic objectives and the subsidiary work in their support, and reflect the role of simulation as a dominant technique. The functions of each part and the relations among them are indicated in the technical summary, Part 2.

The following organizations have participated directly in the program:

| Organization | Contract No. | Time of Performance |
|---|---|---|
| University of Chicago | AF33(038)-15068 Supplements 2 and 11 | 1 Feb. '51-31 Aug. '54 |
| J. B. Rea Company | AF33(038)-15068 Subcontract 2 | 1 Feb. '51-31 Oct. '52 |
| Cook Research Laboratories | AF33(038)-15068 Subcontracts 3 and 9 | 1 Feb. '51-31 May '54 |
| RCA Laboratories | AF33(038)-15068 Subcontract 4 | 1 Feb. '51-1 Mar. '53 |
| Armour Res. Foundation of Ill. Inst. of Technology | AF33(038)-15068 Subcontract 5 | 1 Feb. '52-30 Nov. '52 |
| Northwestern University, Aerial Meas. Lab. | AF33(038)-15068 Subcontract 8 | 17 July '52-22 Aug. '52 |
| Mass. Inst. of Technology, Flight Control Lab. | AF33(038)-15068 Purchase Order A2086 | 20 Apr. '54-31 Aug. '54 |
| Mass. Inst. of Technology, Dynamic Analysis & Control Laboratory | AF33(038)-15068 Purchase Order A23883 | 22 July '53-30 Nov. '53 |
| Mass. Inst. of Technology, D.A.C.L. | AF33(616)-2263 Task Statement 2 | 1 Dec. '53-30 Sept. '54 |
| Nat. Bur. of Standards Corona, which became | AF33(038)-51-4345-E | 25 Feb. '51-Sept. '53 |
| Naval Ordnance Lab., Corona | MIPR(33-616)54-154 | 20 Nov. '53-31 Dec. '55 |

This is a record of formal participation only; the program was aided immeasurably by the splendid cooperation of all governmental, industrial and educational organizations (particularly the simulation laboratories) contacted. Although it is impractical to mention them all here, the extent of their assistance is evident throughout the reports and is hereby gratefully acknowledged. Details of these affiliations, including statements of work, may be found throughout the 21 Bimonthly Progress Reports issued by the University of Chicago during the course of the work. (All formal participation in the program is recorded above; missing supplement and subcontract numbers do not pertain to this project.)

The University of Chicago was assigned prime responsibility for integration of the program. This has been effected by a full time staff at the University, and by aperiodic meetings of the following advisory committee, selected by the Air Force:

| | | |
|---|---|---|
| Dean Walter Bartky, Chairman | University of Chicago | 1 Feb. '51-31 Aug. '54 |
| Prof. C. S. Draper | Mass.Inst. of Tech. | 1 Feb. '51-28 Feb. '53 |
| Mr. Donald McDonald | Cook Research Lab. | 1 Feb. '51-31 Aug. '54 |
| Prof. F. J. Murray | Columbia University | 1 Apr. '52-31 Aug. '54 |
| Dr. J. B. Rea | J. B. Rea Company | 1 Feb. '51-28 Feb. '53 |
| Prof. R. C. Seamans, Jr. | Mass.Inst. of Tech. | 1 Sept. '53-31 Aug. '54 |
| Mr. R. J. Shank | Hughes Aircraft Co. | 1 July '51-31 Aug. '54 |
| Dr. H. K. Skramstad | NBS-NOLC | 1 Feb. '51-31 Aug. '54 |
| Mr. A. W. Vance | RCA Laboratories | 1 Feb. '51-31 Aug. '54 |
| ex officio: | | |
| Mr. P. W. Nosker, Project Eng. | WADC | 1 Feb. '51-31 Aug. '54 |
| Dr. B. E. Howard, Secretary | University of Chicago | 1 Feb. '51-31 Aug. '54 |

The meetings have been recorded in the Bimonthly Progress Reports previously mentioned. Except for Dr. Skramstad, who has participated through direct arrangement between NBS-NOLC and WADC, members of the advisory committee who are not connected directly with the University have participated in the program through consulting agreements with the University of Chicago. In addition, similar consulting agreements with the University have provided for the participation of:

| | | |
|---|---|---|
| Dr. R. R. Bennett | Hughes Aircraft Co. | 1 Jan. '52-31 Jan. '54 |
| Mr. J. P. Corbett | Libertyville, Ill. (formerly with the University) | 11 May '54-31 Aug. '54 |
| Mr. G. L. Landsman | Motorola, Inc. | 1 May '54-31 Aug. '54 |
| Dr. Thornton Page | Johns Hopkins Univ. (formerly with the University, and Secretary to the Board until 1 Aug. 51) | 7 Aug. '51-1 Mar. '53 |

| | | |
|---|---|---|
| Prof. M. Z. Krzywoblocki | Univ. of Illinois | 15 Jan. '52-31 Aug. '54 |
| Prof. K. S. Miller | New York Univ. | 2 Nov. '53-31 Aug. '54 |
| Dr. J. Winson | Riverside, N. Y. (formerly consultant to Project Cyclone) | 1 Mar. '53-30 June '54 |

Many others have contributed significantly to the progress of the work. Among those from other organizations in regular attendance at most of the meetings of the committee have been Mr. Charles F. West, Air Force Missile Test Center; Prof. L. L. Rauch, University of Michigan, representing Arnold Engineering Development Center; Col. A. I. Lingard, WADC; and Dr. F. W. Bubb, WADC.

Coordination of the program and administration of the prime contract at the University of Chicago have been under the charge of Dr. Walter Bartky, Dean of the Division of Physical Sciences and Director of the Institute for Air Weapons Research; Dr. B. E. Howard, Assistant to the Director; and Messrs. William R. Allen and William J. Riordan, Group Leaders. The work at the cooperating institutions has been directed by the appropriate member of the advisory committee and his assistants: Dr. H. K. Skramstad and Mr. Gerald L. Landsman at the National Bureau of Standards-Naval Ordnance Laboratory, Corona; Messrs. Donald McDonald and Jay Warshawsky at Cook Research Laboratories; Messrs. A. W. Vance, J. Lehman, and Dr. E. C. Hutter at RCA Laboratories; Dr. J. B. Rea at J. B. Rea Company; Prof. R. C. Seamans at the Flight Control Laboratory and Dr. W. W. Seifert and Mr. H. E. Blanton at the Dynamic Analysis and Control Laboratory, Mass. Inst. of Technology. V. H. Disney, S. Hori, and G. F. Warnke at Armour Research Foundation and J. C. MacAnulty and George Goelz at Northwestern University, Aerial Measurements Lab. have directed the contributory studies at their respective organizations. More explicit credit is found in appropriate places throughout the reports; biographical sketches are in Part 1. Space does not allow full credit that is due to all the workers on the combined project, but special mention is certainly due the project engineer for his conception of the project and for his cooperation during its execution.

Except for Appendix 3, this volume was written in its entirity by C. D. LaBudde, and prepared for publication by E. R. Spangler at the University of Chicago. Acknowledgement is gratefully made for the cooperation of the many computing facilities throughout the nation in making the survey of recording methods on which this volume is based.

## ABSTRACT

The recording needs of a system dynamics laboratory fall into four general categories, determined by the types of problems solved by the facility. The first category requires a recorder capable of making a visual display of a large amount of output data. Accuracy requirements are low here, and it is required that only the form and stability of the functions are preserved. The second category requires a high accuracy recorder to make a visual display of output data having the form of voltage levels. The amount of output data here will be relatively small. The third category requires a recorder capable of plotting one voltage level with respect to another. The amount of data required in this form will also be small. The fourth category requires a recorder which will record voltage levels in digital form suitable for use in numerical computations. High accuracy is required and the amount of output is relatively small. The first three recording requirements can be met by commercially available equipment. The fourth requirement can be met by existing recording techniques, but the actual equipment is not commercially available.

## PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

Aldro Lingard

ALDRO LINGARD
Colonel, USAF
Chief, Aeronautical Research Laboratory
Directorate of Research

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# 1. THE RECORDER PROBLEM

The problems of data recording are largely those arising from a need to transform information into forms susceptible to convenient use by the scientist in research. Formerly data recording was manual, but modern data recording requirements in speed, accuracy, and volume are so enormous that special purpose recording devices must be constructed to accomplish the task. The modern insurance company offers an example of the contemporary needs in recording data, but the greatest impetus to construct better recorders has been provided by the growth and development of large scale computers. So great has been the influence of computers on recorders that the tendency in the computer field is to regard recorder problems as subordinate to those of computers. But the problems of data recording and presentation are distinct from those of computers and actually are more related to those of procedures. If, for example, the recorder used in a problem makes a graphical plot of the solution and if it is required that the solution be in numerical form, a great deal of labor is necessary to "read" the graph and to make the conversion by manual means. For data of considerable size, this means of conversion becomes far too slow and subject to human errors. Thus, in this case, it would be necessary to have a recorder which could present the data numerically so that it could be used directly by the scientist without intermediate processing.

We may define the problem precisely in terms of Prof. Murray's definition of a mathematical machine (Reference 1). "A mathematical machine is a mechanism which provides information concerning the relationships in a specified set of mathematical concepts. This requires that the machine contain a realization of the set of mathematical concepts." For our purposes we shall regard that part of the machine which realizes the mathematical concepts as the computer and that part of the machine which conveys information to the operator as the recorder. In practice these pieces of equipment are usually distinct and physically separated from each other.

The choice of a recorder will, of course, depend upon the computer and the form of the data desired. But another important part of the form of the data displayed by the recorder is the error introduced into the solution by the recorder system itself. For example, the error introduced by the recorder

Manuscript released by the author in August 1954 for publication as a WADC Technical Report.

may be so great as to render the data displayed by the recorder useless for a given application, even though the data may be in the desired form. Thus we must take into account three elements in the selection of the recorders for our particular problems: the computer, the form of the data required, and the error introduced by the recording system.

The dynamic system facility solves problems relating to the motion of air vehicles by means of large scale electronic analog computers. The solutions or output data from these computers consist of functions of time represented as DC voltage levels. The recorders for these computers are required to make visual displays in such a manner as to be useful to the dynamic system engineer and will vary according to the problem solved. In general, the problems solved by a dynamic system facility may be divided into four categories according to the type of display required, each category using a different type of recorder.

The first type of problem uses a large number of computer runs, and obtains a large number of solutions for variations of the input parameters over large ranges. The purpose is to make rough visual comparisons of the solutions as to form and stability. What is required is a form of output which will display the functions as graphs, and the form should be able to accommodate a large amount of output data. Relatively large errors in the form of simple time lags and amplitude multiplications can be tolerated here; the errors, however, should not introduce fundamental changes in the form or stability of the solutions.

The second type of problem uses a relatively small number of computer runs to obtain solutions for small, critical ranges of the input parameters in order to compare in greater detail and refinement the different solutions obtained. The accuracy required here will be relatively high; the error must be such as to keep the recorded function within a fixed preassigned percentage of full scale of the theoretical solution. The amount of output will be relatively small, and graphical means are satisfactory here.

The third type of problem consists of making computer runs with the purpose of comparing one or two output functions with respect to another. This form of recording is primarily a convenience; the information obtained here could be derived from other means of recording, but only after a good deal of laborious computation. The amount of output data here will be small but the accuracy requirements are relatively high, and the recorded function should be within a certain preassigned percentage of full scale of the theoretical solution.

The fourth type of problem consists of making computer runs to obtain solutions for extensive mathematical analysis, e.g., frequency analysis, correlation analysis with other functions of time, subtraction from other functions to obtain difference functions, etc. The output data must be very accurately recorded and in such a form that they may be inserted into a digital computer, or used by an operator of a hand calculator for further computation. This necessitates the data's being converted into digital form and printed as numbers on paper or stored in some medium which can be used as input to a digital computer.

These, then, are the general types of problems solved at a dynamic system facility as viewed from the point of view of recorders. Precise requirements for each type of recorder and how these requirements should best be met are set forth in the succeeding chapters.

## 2. SPECIFICATIONS

The specifications described here have been determined with the operating and mathematical requirements of the dynamic systems laboratory primarily in mind. No attempt has been made to determine precise engineering specifications for the equipment; the Advisory Board on Simulation is not committed to any particular engineering design, it being felt that engineering specifications are best determined by the organization actually undertaking the design, development, and construction of the recording equipment to meet requirements set forth in this chapter. It is believed, however, that accuracy and speed requirements will necessitate the use of electronic, rather than electromechanical, components wherever possible in the recording equipment.

All recorders should make a visual display of DC voltages which represent functions of time developed in analog computers. The DC voltages must range from -100 to +100 volts and contain frequencies from 0 to 25 cps full scale. The input impedance of the recorders should be sufficiently high to draw a negligible amount of current from the computer or to introduce no appreciable error into the computing system. The exact figure for this impedance can be determined from the characteristics of the dynamic systems synthesizer. The recorders should be packaged as separate units with their own power supplies, and mounted on casters so that they may easily be moved about. Connecting leads from the recorders to the computers should be of the plug-in jack type for ease of connection and disconnection.

The recorders should display the necessary DC voltages in four different ways, each of which is described below.

The first type of recording should be designed to display graphically as functions of time a large amount of output data in the form of DC voltages from analog computers. The primary purpose here is to determine the general form and stability of the functions being recorded. High accuracy is not of primary importance; 5% of full scale is sufficient for this purpose. Because the amount of data to be recorded this way will be large, the recording medium should be as inexpensive as possible, which would imply that the recording be done by means of ink on paper. The paper width should be about five centimeters for each channel. If possible, the recording should be accomplished in rectilinear coordinates, although this requirement is not absolutely necessary. The coordinate lines of the coordinate system used should be printed on the recording paper. For convenience in reading a large amount of data, each channel should have provision for the recording of more than one variable but not more than three. There should be provision for a timing pulse marker on one channel of each set of channels being recorded simultaneously, and means should be provided where variables are being recorded on several channels simultaneously to assure that values of all variables corresponding to a given time are recorded on the same abscissa line. The graph should be in full view of the operator as it is being recorded, and the paper should be mounted horizontally on a translucent surface above a light source so that several graphs can be overlayed for comparison. The recording paper should be accessible so that notations can be made on the paper during the process of recording. The paper can be rolled on a take-up roller providing provision is made for tearing the paper off at any point. The paper speed should be controlled to maintain the accuracy figure, and there should be provision for at least six paper speeds, ranging from about one millimeter per second to about 5-10 centimeters per second.

The second form of recording should be designed to display graphically as functions of time a relatively small amount of output data consisting of DC voltages from analog computers at relatively high accuracy. The primary purpose of this recording is to obtain precise graphical information about certain portions of the output data which are thought to be critical. The overall accuracy figure should be 1% of full scale. The recording should be accomplished on paper which is at least five centimeters wide for each channel. For convenience at least two but not more than three variables should be recorded on each channel. Provision should be made for a timing pulse marker on one channel of each set of channels being recorded simultaneously. It is not necessary here to see the graph as it is being traced or to have immediate access to the graph after it is prepared, thus permitting the use of photographic paper and a light beam as the means of recording at the required frequencies and accuracy. The recording should be accomplished in rectilinear coordinates with the coordinate lines printed upon the recording paper. Means should be provided such that when variables are being recorded on several channels simultaneously values of all variables corresponding to a given time are recorded on the same abscissa line. Provision should be made for several paper speeds, ranging from about one centimeter per second to 50 or more centimeters per second.

The third form of recording should be designed to graph one function of time with respect to another function of time represented by voltages in an analog computer. The amount of data recorded this way will be small and the precision 1% of full scale. The recording should be done on graph paper at least two feet square, and in rectilinear coordinates. The frequency response requirements are not as high as those for the first two forms of recording: five cps for amplitudes of one inch or less. This will permit the recording by pen and ink on ordinary paper. This paper should be in full view of the operator so that the trace may be viewed while it is being made. The recording paper should be mounted horizontally on translucent surface over a light so that several graphs may be superimposed and read simultaneously. There should be provision for recording at least two functions of time with respect to another on the same graph paper.

The fourth form of recording should be designed to convert DC voltages to a sequence of digital numbers which can either be inserted into a digital computer for further computation or read out on some suitable output device. The precision of the conversion should be 0.1%; or the voltage readings each should be converted into the equivalent of three decimal digits plus sign. The method of sampling the voltage should be uniform, that is, there should exist an interval of time such that if the entire recording time is divided uniformly into intervals of this length, one and only one sample will occur in each interval. The length of this interval should be no greater than .01 seconds, which implies a sampling rate of 100 samples per second. The time required to read a sample should not be greater than six microseconds. The numbers representing the voltage levels should be stored as digital information on magnetic tape. The speed of the magnetic tape for read-in should be the equivalent of 1100 binary digits per second. Provision should be made for an output printer operating from the magnetic tape which prints the time and the value of the voltage corresponding to the time as follows:

| Time | Value |
|---|---|
| .01 | ± xx.x |
| .02 | ± xx.x |
| .03 | ± xx.x |

The information on the magnetic tape should be such that the tape may be used as input to a suitable digital computer. There should be provision for multiplexing of up to three analog voltages on one digital output channel.

All four recording systems should be packaged as conveniently as possible to allow changeover from one system to another without the use of tools. All normal adjustments on the recorders, such as paper speed changes, gain controls, etc., should be made without the use of tools or other auxiliary equipment. All recorders should have provision for checking their operation and for as easy maintenance and repair as possible.

## 3. CONCLUSIONS AND RECOMMENDATIONS

At the present time in an analog computation system, the recorder contributes more error to the system than any other single component. Consequently a dynamic systems laboratory has a definite interest in the further improvement of recorders for simulation purposes, although its function is not primarily development. In the meantime, however, where commercially available recorders prove adequate for the task at hand, they should be used by a laboratory although the following series of requisites must be met with respect to recorder requirements. These recommendations are based largely on the information gathered in the survey of recorders. (See Chapter 4.)

The requirements of the first form of recording described in Chapter 2, are met by Brush Development and Midcentury Instruments. At least two six-channel units and several one- or two-channel units are required for dynamic systems facility purposes. The curvilinear coordinate system of these recorders constitutes a disadvantage, but this feature is not critical at the accuracies of these recorders. A dynamic systems facility has an interest in the improvement of the accuracy and frequency response of these recorders, but should not itself initiate by contract any such program. The Brush or Midcentury recorders as presently designed should prove adequate for most dynamic systems applications.

The requirements of the second form of recording are met by an oscillograph of the kind manufactured by Consolidated Engineering Corporation or Heiland Research Corporation. It should be noted, however, that these devices are used to record small voltage levels, as compared to those in an analog computer, and that the input impedance to these devices is too low to be used with ordinary analog computer operational amplifiers. Any contract to acquire one of these recorders from either corporation should include the stipulation that the company make the necessary impedance adjustments so that voltages from -100 to +100 may be recorded and so that the input impedance is sufficiently high for the dynamic systems synthesizer.

The requirements of the third form of recording, are met by the Electronics Associates Plotting Board. These plotting boards have proved satisfactory in the past in simulation problems.

No existent commercially available device can meet the requirements set by the fourth form of recording, although several existing designs and one-of-a-kind machines may well meet the needs of a facility, with only slight modifications. To obtain such a device, a dynamic systems facility should let a study and development contract to a suitable organization. Although the specifications in Chapter 2 are purely mathematical and the Advisory Board on Simulation is not committeed a priori to any particular engineering design, it is felt that the differential counting coder referred to as the binary quantizer, developed by K. H. Barney and described in Appendix 3 and references 2 and 3, offers the best hope of meeting both the requirements of uniform sampling and very small sampling time. It is recommended that the contractor attempt to develop an analog-to-digital converter based on this principle, although other workable designs should not immediately be ruled out. Some care must be given to the problem of intermediate storage for the numbers representing converted voltages. Despite the fact that speed and frequency requirements necessitate the recording of the numbers on magnetic tape, it may be necessary or convenient to have equipment capable of reading the data from magnetic tape and converting it to data on punched tape, punched cards, or some intermediate, slower storage before attempting to read the data out on a printer or feed it into a digital computer.

To test the recording equipment for performance and accuracy, the following procedures are recommended for use in the facility. A constant voltage may be recorded for some time to determine drift and noise characteristics of the recorder. Sine waves at various frequencies, generated by a precision sine wave generator, may be recorded with the recorder set for full scale deflection, to determine frequency response. In the Brush or Midcentury recorders, the paper speeds should be tested by means of the timing pulses to assure that the variation in the speeds as determined by the intervals between the recorded timing pulses does not make the recorded graph exceed the required accuracy figure.

# 4. SURVEY OF RECORDERS

The following is a summary of the survey made of existent recording systems and their characteristics as listed in the recorder check list. The purpose of the survey was to obtain information concerning the present state of the recorder field and its relation to system dynamics recorder requirements. In the survey, 185 manufacturers of recording equipment were reached by letter or visit and information was obtained concerning their equipment.

Analysis of the data indicates that four types of recording systems meet the requirements established in the specifications for the four categories of problems solved by a laboratory. Of these, three are commercially available, and a working model has been constructed of the fourth. The three commercially available recording devices of interest to a laboratory are the recorders manufactured by Brush and Midcentury, the oscillographs manufactured by Heiland and Consolidated Engineering, and the plotting boards manufactured by Electronics Associates. The fourth type of recording system of interest to the laboratory is the analog to digital converter designed by Barney and constructed at Northwestern University. Currently studies are under way at the Ford Instrument Company concerning the feasibility of such a device, and plans are under consideration for constructing such a device. Copies of this device may become available for purchase by other agencies.

The Brush and Midcentury recorders have very similar characteristics, both using the same type of pen-driving device. The recorders record a voltage level as a function of time on a three inch wide strip of paper. The voltage is recorded in curvilinear coordinates, which is the chief drawback of these devices. The amplitude response is flat out to 30 cycles for full scale deflection. The chart speeds that are available are three in each unit; different sets of three speeds are available with different choices of motors and sprockets. The accuracy is of a relatively low order (1% - 5%) of full scale, and these devices are best suited for large amounts of output data, as in the first category of problems solved by a laboratory.

The Heiland and Consolidated Engineering oscillographs are photographic recording devices that record voltage levels as functions of time by means of light beams on photographically sensitive paper. The accuracy of these devices is within .1% of full scale, which is approximately two inches. The frequency response is good out to 100 cycles or better. There is, however, the drawback that these devices are designed to measure small voltage displacements; the impedance matching difficulties have to be met before voltages of up to 100 volts can be recorded. Multiple channel units are available, and four paper speeds are available in one unit. Other speeds are available with other motors and gear ratios. These devices meet the requirements set forth in the second category of problems solved by a system dynamics laboratory.

The Electronics Associates plotting board records one voltage as a function of another on paper approximately 2.5 feet square. The static accuracy of this device is approximately .05% of full scale, and the total error is approximately .1%. The frequency response can best be given in terms of the pen accelerations, which are 150 in/sec$^2$ maximum forward and 350 in/sec$^2$ maximum laterally. Maximum usable writing speed is approximately 8.5 inch per second. Two channels of plotting are available on one unit.

The Ford Instrument Co. will make information available on its devices as the studies progress. As yet, no information has been formally released. See Appendix 3 for a more detailed description of this device.

## REFERENCES


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2. Hollander, G. L., "Criteria for the Selection of Analogue-to-Digital Converters," paper presented at the 1953 National Electronics Conference in Chicago.

3. Bower, C. G., Survey of Analogue-to-Digital Converters, National Bureau of Standards Report 2755, July 1953.

4. Oliver, B. M., J. R. Pierce, C. E. Shannon, "The Philosophy of P.C.M.," Proceedings, I.R.E., xxxvi (Nov. 1948), 1324-1331.

5. Jackson, D., "The Theory of Approximation," American Mathematical Society Colloquiem Series, Volume 11, New York, 1930.

Appendix 1

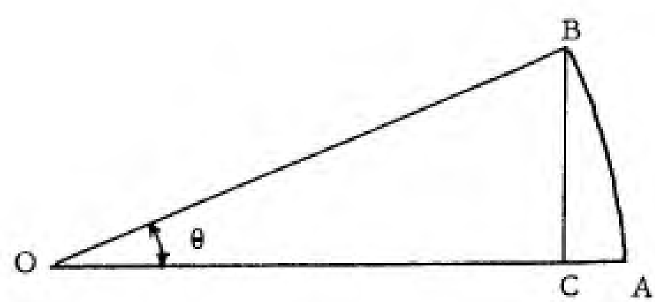
## ON THE MATHEMATICS OF BRUSH AND SANBORN RECORDERS

It is well known that the curvilinear recording mechanism of the Brush and Sanborn recorders produces a distorted representation of the function being recorded. The purpose of this appendix is to derive exact mathematical expressions for this distortion.

If $F(x,t)$ is a function of two variables, the equation $F(x,t) = 0$ will define a function $x = f(t)$ under very general conditions. Consider $(t,x)$ to be rectilinear coordinates in the plane and assume $F(x,t) = 0$ is being traced out by a Brush recorder. The graph of $F(x,t) = 0$ will have the equation $G(x_1,t_1) = 0$ where $(t_1,x_1)$ are rectilinear coordinates on the Brush recording paper $G(x_1,t_1) = 0$ will define a function $x_1 = g(t_1)$. The equations relating the coordinates are:

$$
\begin{aligned}
t_1 &= t - L + L \cos k \frac{x}{L} \\
x_1 &= L \sin k \frac{x}{L} \\
t &= t_1 + L - \sqrt{L^2 - x_1^2} \\
x &= \frac{L}{k} \sin^{-1} \frac{x_1}{L}
\end{aligned}
\tag{1.1}
$$

where L is the length of the Brush recording pen from pivot point to tip and k is a scaling factor. These equations may be easily derived from the geometry of the Brush recorder as seen in the following diagram.



Let O be the pivot point of the pen and let A,B represent pen point positions. Then OB = OA = L. If a displacement x is to be recorded at a time t, then the arc length AB will be proportional to x, that is, $L\theta = kx$.

Let O be the pivot point of the pen and let A,B represent pen point positions. Then OB = OA = L. If a displacement x is to be recorded at a time t, then the arc length $\widehat{AB}$ will be proportional to x, that is, $L\theta = kx$. The coordinates $(t_1, x_1)$ of B are given by $(\overline{OC}, \overline{CB})$. But $\overline{OC} = \overline{OA} - \overline{CA} = t - (L - L\cos\frac{kx}{L})$ and $\overline{CB} = L\sin\frac{kx}{L}$. Hence equations (1.1) follow. The following equations are included for use later.

$$\frac{\partial t_1}{\partial t} = 1$$

$$\frac{\partial t_1}{\partial x} = k\sin\frac{kx}{L} = -\frac{kx_1}{L}$$

$$\frac{\partial x_1}{\partial t} = 0 \tag{1.2}$$

$$\frac{\partial x_1}{\partial x} = k\cos\frac{kx}{L} = \frac{k}{L}\sqrt{L^2 - x_1^{\,2}}$$

Now if the function $x = f(t)$ is represented by a graph on the Brush recorder given by $x_1 = g(t_1)$, the equation $f'(t) = g'(t_1)$ will not hold true in general, at corresponding points; but the actual value of the derivative is related to the corresponding measured value of the derivative by a simple equation derived below.

Now

$$f' = -\frac{\partial F/\partial t}{\partial F/\partial x} \qquad \text{and} \qquad g' = -\frac{\partial G/\partial t_1}{\partial G/\partial x_1}$$

and

$$\frac{\partial F}{\partial t} = \frac{\partial G}{\partial t_1}\frac{\partial t_1}{\partial t} + \frac{\partial G}{\partial x_1}\frac{\partial x_1}{\partial t} \; ; \quad \frac{\partial F}{\partial x} = \frac{\partial G}{\partial t_1}\frac{\partial t_1}{\partial x} + \frac{\partial G}{\partial x_1}\frac{\partial x_1}{\partial x}$$

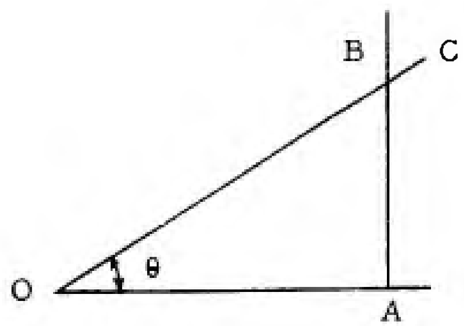
$$\text{so} \quad f' = -\frac{(\partial G/\partial t_1)(\partial t_1/\partial t) + (\partial G/\partial x_1)(\partial x_1/\partial t)}{(\partial G/\partial t_1)(\partial t_1/\partial x) + (\partial G/\partial x_1)(\partial x_1/\partial x)} = -\frac{g'(\partial t_1/\partial t) - \partial x_1/\partial t}{g'(\partial t_1/\partial x) - \partial x_1/\partial x} \; ;$$

substituting from (1.2) we get

$$f'(t) = \frac{g'(t_1)}{k\left[g'(t_1)\sin\frac{kx}{L} + \cos\frac{kx}{L}\right]} = \frac{L\, g'(t_1)}{k\left[g'(t_1)\, x_1 + \sqrt{L^2 - x_1^2}\right]} \qquad (1.3)$$

This enables one to compute the value of the actual derivative $f'(t)$ in terms of the measured derivative $g'(t_1)$ on the Brush recording paper. In practice the $(t,x)$ coordinate lines are printed on the Brush recording paper so that the use of the $(t_1, x_1)$ coordinates for reading purposes is largely unnecessary. However, in determining derivatives, it may be more convenient to measure $x_1$ with a ruler, than to determine trigonometric functions of $\frac{kx}{L}$.

The Sanborn recorder uses a Brush type pen motor drive with the pen arm sliding across a fixed "hot" wire. Although the recording appears to be in rectilinear, it is in reality not, because the angular rotation of the pen arm is proportional to the input voltage. The following diagram gives the geometry of the Sanborn recorder.




Let O be the pivot point, AB the "hot wire", and OC the pen arm. Let OA = $\lambda$, a fixed constant. Then OC = L as before, $L\theta = kx$, $AB = \lambda \tan\frac{kx}{L}$. Then the relations between the input variables $(t,x)$ and the Sanborn plotting paper coordinates $(t_1, x_1)$ are as follows.

$$t_1 = t$$

$$x_1 = \lambda \tan\frac{kx}{L} \qquad (1.4)$$

As before we can compute the equation relating the actual value of the derivative $f'(t)$ with the measured one $g'(t_1)$ on the Sanborn recorder.

$$f' = - \frac{g'(\partial t_1/\partial t) - \partial x_1/\partial t}{g'(\partial t_1/\partial x) - \partial x_1/\partial x}$$

$$f'(t) = + \frac{L}{\lambda k} \; g'(t_1) \cos^2 \frac{kx}{L} = \frac{L}{k} \; g'(t_1) \frac{\lambda}{\lambda^2 + x_1^2} \tag{1.5}$$

Appendix 2

## ON ANALOG TO DIGITAL CONVERSION

This appendix is a discussion of the important parameters in analog to digital converters and the relations among them. These relations impose limitations upon analog to digital converters, and were used earlier to aid in determining specifications. Most of these relations are neither profound nor original with this author, but it is useful to have them compiled in one place for the purposes of evaluation of analog to digital converters. We make the following definitions:

| | | |
|---|---|---|
| $f(t)$ | = | input analog quantity to be digitized |
| $n$ | = | total number of cycles |
| $\omega$ | = | frequency (radian/second) |
| $T$ | = | time to read one sample |
| $R$ | = | time between initiation of samples |
| $p$ | = | precision of digitizer |
| $P$ | = | pulse rate (where applicable) |
| $M$ | = | number of quantized levels in digitizer |
| $L$ | = | total length of time for conversion |
| $a_n, b_n$ | = | fourier coefficients for $f(t)$ in $[0,L]$ |
| $A$ | = | $\sup_t \lvert f(t)\rvert$ |
| $V$ | = | $\sup_t \lvert f'(t)\rvert$ |
| $\Omega$ | = | $\sup_\omega \lvert\omega\rvert$ (assumed to be finite) |
| $N$ | = | $\sup_n \lvert n\rvert$ (assumed to be finite) |
| $T_i$ | = | $\inf_T T$; $T_a = \sup_T T$ |
| $R_i$ | = | $\inf_R R$; $R_a = \sup_R R$ |
| $a$ | = | $\sup_{a_n, b_n} (\lvert a_n\rvert, \lvert b_n\rvert)$ |

The following equations hold true by definition:

$$f(t) = a_o + \sum_{n=1}^{N} a_n \cos \frac{2\pi\, nt}{L} + \sum_{n=1}^{N} b_n \sin \frac{2\pi\, nt}{L} \qquad (2.1)$$

$$a_n = \frac{L}{2} \int_0^L \cos \frac{2\pi\, nt}{L} f(t)dt; \quad b_n = \frac{L}{2} \int_0^L \sin \frac{2\pi\, nt}{L} f(t)dt \qquad (2.2)$$

$$\omega L = 2\pi n; \qquad \Omega L = 2\pi N \qquad (2.3)$$

The following is a consequence of Shannon's theorem (see Reference 4).

$$2NR_a \leq L \qquad (2.4)$$

This relation follows from Bernstein's theorem (see Reference 5).

$$V \leq \Omega A \qquad (2.5)$$

The following are easy consequences of the definitions:

$$T_a \leq R_i \qquad (2.6)$$

$$2A = pM \qquad (2.7)$$

$$a \leq \frac{AL}{2} \qquad (2.8)$$

The following relations are easy consequences of Rolle's theorem:

$$VT_a \leq pA; \qquad (2.9)$$

$$\Omega T_a \leq p \qquad (2.9a)$$

(2.9a) is derived from (2.9) by letting $f(t) = A \sin \Omega t$.

If f(t) is converted into a sequence of pulse trains each of whose length is proportional to the value of f(t), then the following relation holds:

$$V \leq pAP \tag{2.10}$$

For example, if given p = .001, A = 100, N/L = 25 cycles per second, then, applying (2.4), we have

$R_a \leq .02$ seconds ; applying (2.3), (2.5) we have

$V \leq 5000\pi$ ; applying (2.9), (2.9a) we have

$T_a \leq \frac{2}{\pi} \times 10^{-5}$ seconds ; and applying (2.10) we have

$P \geq 50{,}000\pi$ pulses per second.

In practice these relations give the best possible values for the variables concerned. For example, V can never exceed $\Omega A$, and in reality it will always be considerably less. These relations may best be used to exclude impossible cases rather than to predict feasible cases.

Further information on analog-to-digital conversions in general is contained in "Criteria for the Selection of Analog-to-Digital Converters" by Gerhard L. Hollander. This is an appendix to an MIT thesis (Servomechanism Laboratory) and was presented as a paper at the National Electronics Conference in Chicago on 29 September 1953.

Appendix 3*

# AN ANALOG TO DIGITAL CONVERSION TECHNIQUE

by

J. P. Corbett

This appendix is a description of an analog to digital conversion device designed and constructed by K. H. Barney and described in his thesis at Northwestern University in 1947.** This device is considered one of the most promising for meeting dynamic systems engineering requirements.

The Barney device consists of an analog to digital - digital to analog feedback loop as diagrammed in the figure. The digital portion of the loop consists of a counter which can be made to run either backward or forward, depending on whether a signal from a comparison device is positive or negative.

Associated with this counter is a storage register which can be periodically triggered to retain the digits contained in the counter at the instant of triggering. The purpose of this register is to permit readout from the counter at a rate slower than the counting rate.

The digital-to-analog conversion is accomplished by slaving a voltage gate to each of the digital counter elements. If these gates corresponding to the binary number

$$a_1 a_2 \ldots\ldots a_n$$

are operated in such a manner that the k th gate has a voltage output of

$$\begin{array}{ll} 2^{-K} E & \text{if} \quad a_K = 1 \\ 0 & \text{if} \quad a_K = 0 \end{array}$$

---

*From Advisory Board on Simulation Technical Note 45, 11 June 1954.

** See also K. H. Barney, "The Binary Quantizer," Elec. Eng., lxviii (1949), 1053-1058.

the sum of the output voltages is equal to E times the number represented by the state of the counter. If a suitable bias voltage is subtracted from the sum of the outputs of the voltage gates (one half the maximum value), we have a zero centered voltage scale for the output of the digital to analog device. The scale of the device may be chosen at pleasure by changing the level of the gate voltage E.

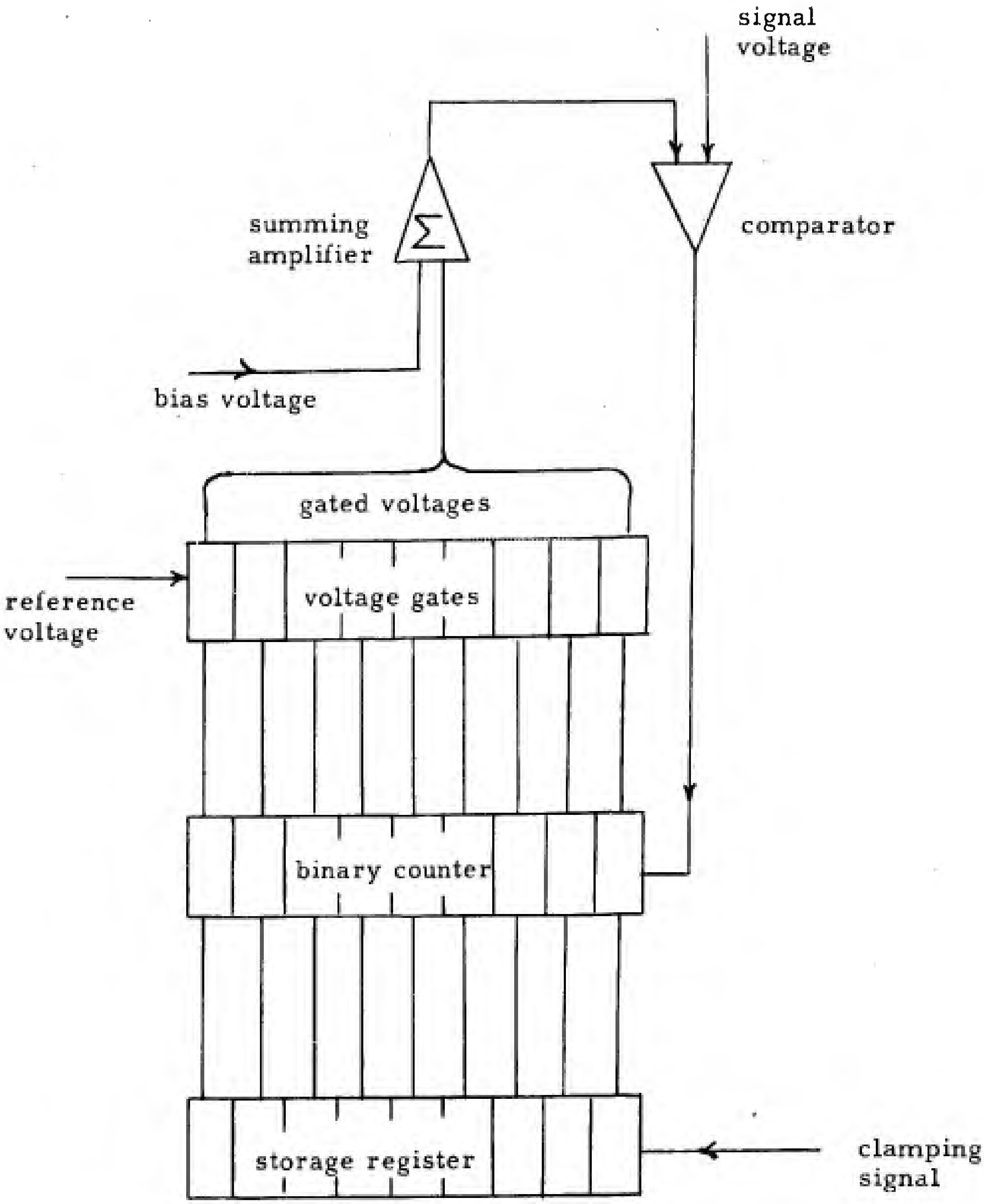
The output of these gates is now compared in a summing device with the voltage whose digital representation is desired. Whether the output of the comparison device is positive or negative determines the direction in which the counter will run. If the gates were ideal devices in the sense that their rise time could be neglected, and if the rate of signal voltage increase (measured in counter units per second) does not exceed the counting rate, the device would produce a result differing at most by the smallest voltage step $2^{-n}$ E.

The voltage rate is $10^4$ volts per second, which corresponds to a counter rate of about $10^5$ units per second. This is a 10 microsecond counting interval, and counters of much higher rates are available. This last calculation is based on the use of an 11 gate counter, the smallest voltage gated being 0.1 volt, the remaining gates gating this voltage multiplied by the appropriate power of two. On this basis an 11 gate device would have a range of just over plus or minus 100 volts.

Rise time of the gates is a serious limitation of this device. In Barney's prototype device a one-quarter megacycle counter was used, but only four binary digits were provided. His test results are therefore not conclusive for our purposes. It seems that a three to four microsecond effective gating time is possible and that this rather than counter speed would be a limiting factor.

A final serious limitation is precision of the gating levels. A considerable problem would be encountered in adjusting the individual gating levels, and there would be some question of the stability of this adjustment. The tolerance required on the individual gated voltages would have to be something less than .01 volt so that the total error from the sum of the 11 gate device would be less than 0.1 volt.

In summing up, it seems that there is definite promise in a device of this sort. Without some definite experiments on components, however, it is not possible to predict the attainable precision.




Schematic Representation of the Analog to Digital Converter

Appendix 4

# RECORDER CHECK LIST

What follows is a check list of what are considered to be the important parameters in recording systems. In most cases not all of the parameters in the list were obtained or obtainable in the course of the survey for each recorder, but partial answers to most of the items enabled the survey to establish in all cases at least a cursory knowledge of the equipment under survey.

## RECORDER CHECK LIST

I. General Characteristics

- A. Size
- B. Weight
- C. Cost
- D. Delivery Time
- E. Power Requirements
- F. Special Requirements (air conditioning, etc.)
- G. Number of units in existence

II. Characteristics of Inputs to the Equipment

- A. Analog inputs
  1. Number of and description of each kind of input (AC or DC voltage levels, shaft rotation, etc.) receivable by the equipment
  2. Maximum displacements, first derivatives, and second derivatives of each input receivable by the equipment
- B. Digital inputs
  1. Number of and description of each kind of input (pulses, descrete voltage levels, digital numbers, etc.)
  2. Speed characteristics of each input (maximum pulse rate receivable, maximum number of digits per second receivable, etc.)
  3. Size characteristics of each input (maximum length of digital numbers receivable, form of digital representation, etc.)

III. <u>Characteristics of Outputs or Information-Displaying Devices</u>

A. Analog outputs

1. Number of and description of each kind of output (dial readings, paper recordings, oscilloscope traces, etc.)
2. Maximum displacement, first time derivative, and second time derivative for analog quantity in each output; other speed characteristics such as paper speeds for paper recorders, etc.
3. Accuracy characteristics, such as noise, width of trace on the paper, drift, calibration of the dials, etc. for each output

B. Digital outputs

1. Number of and description of each kind of output (neon lights, registers, paper recorders, etc.)
2. Speed characteristics of each output, such as maximum number of digits recorded per second, maximum number of changes in the register per second, etc.
3. Size characteristics of each output, such as register length, maximum length of digital number recorded, and form of digital representation

IV. <u>Flexibility and Organization</u>

A. Interconnections of outputs or displays and accomplishment of these interconnections

B. Analog input to analog output

1. Amplitude and phase shift characteristics of the output as a function of the frequency of the input
2. General accuracy characteristics (how well does the output reproduce the input?)
3. Method of conversion from one analog quantity to another

C. Analog to digital conversion (analog input to digital output)

1. Method of analog input sampling, sampling rate, method of analog to digital conversion
2. General accuracy characteristics

D. Digital to analog conversion (digital input to analog output)

1. Amplitude and phase shift characteristics of the output as functions of the frequency of sinusoidal input
2. General accuracy characteristics
3. Method of digital to analog conversion

E. Digital input to digital output

1. Method of conversion from one digital quanity to the other
2. Accuracy and source of errors (truncation, roundoff, etc.)

V. Operation and Maintenance

A. Automatic Checking devices, if any
B. Special Maintenance Techniques or equipment required, if any
C. Special operational techniques required, if any

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